Position Accuracy of Aircraft Area Navigation Systems and the Effect of System Parameters

Edwin C. Foudriat*
Marquette University, Milwaukee, Wis.

The steady-state solution to the stochastic differential equation describing the error covariance matrix for a simplified area navigation system has been obtained. The solution shows that nominal error deviations of less than 1.0 nmi are feasible for a DME system. The manner in which estimates are affected by range, air data system accuracy, measurement time interval, and gust deviations, and their impacts on the area navigation system are discussed.

Introduction

INTEGRATED digital hardware have resulted in the development of practical area navigation systems for commercial and private aviation. These systems process VOR/DME radio navigation observations on-line to obtain fixes and can use air data, Doppler, or inertial measurements for dead reckoning between fixes. A critical factor in design and utilization of area navigation systems in the overall air traffic control problem is accuracy.

The static accuracy of the navigation fixes is a geometric problem and easily solved.[1,2] However, for a vehicle in motion, where dead-reckoning system errors are included and fixes are combined in a weighted mean-square estimate, the problem is redefined in terms of a stochastic differential equation. The general solution of this equation is not readily available.

A recent study of an area navigation concept has lead to the development of simple dead-reckoning and measurement-processing equations.[3] In essence, the system utilizes coordinate transformations to define the actual A/C position relative to moving, flat-earth coordinates. The net result of these transformations is a set of dead-reckoning equations which are uncoupled in their "along-track" and "cross-track" modes and linear with respect to each mode. These equations have closed-form general solutions.

By utilizing this format and some additional assumptions described in this paper, solutions to the stochastic differential equation are possible under Gaussian error assumptions. These solutions determine accuracy estimates for the area navigation problem and can be used to study the effects of observation errors, observation rates, and external disturbances. This paper outlines the stochastic differential equation solution and presents an analysis for typical area navigation system conditions.

Stochastic Differential Equation for the Area Navigation Problem

The dead-reckoning system equation for either along- or cross-track modes can be defined by the uncoupled set of differential equations[†3]

$$\dot{y} = v + n + U(t);\ \dot{v} = N_w \tag{1}$$

where y = track position relative to moving coordinate system; v = wind; $U = A/C$ velocity (resolved into its track components); $n = A/C$ velocity error; N_w = wind random error.

The stochastic differential equation for a minimum-error linear estimation system is the matrix Ricatti equation.[4] For a continuous system with discrete observations, two distinct equations are necessary. The equation for propagation between measurement points is

$$P(k+1\,|k) = \Phi(\Delta)P(k|\,k)\Phi^T(\Delta) + S \tag{2}$$

where $P(k\,|j)$ = covariance of position estimation error at time t_k when observation up to t_j are used; $\Phi(\Delta)$ = state transition matrix of Eq. (1) for Δ time interval.

The term S is the result of the system disturbance errors. For uncorrelated velocity and wind random errors, S is

$$S = \begin{bmatrix} q_v\Delta + q_w(\Delta^3/3) & q_w(\Delta^2/2) \\ q_w(\Delta^2/2) & q_w\Delta \end{bmatrix} \tag{3}$$

where q_v and q_w are velocity and wind covariance, respectively.

At the observation point, $t = t_{k+1}$ the covariance matrix takes the form of

$$P(k+1\,|k+1) = P(k+1|\,k) - P(k+1\,|k) \times \underline{H}^T(\underline{H}P\underline{H}^T + R)^{-1}\underline{H}P(k+1\,|k) \tag{4}$$

where R = covariance of the observation error; $\underline{H} = (h,o)$ = observation sensitivity coefficient. For DME and VOR measurement

$$h = y_i/\rho$$

and

$$h = (z_i/y_i^2 + z_i^2)$$

respectively, where ρ = slant range from vehicle to station; y_i = along directional axis distance from vehicle to station; z_i = cross directional axis distance from vehicle to station.

The general solution to Eqs. (2) and (4) contain both transient and forcing terms. An "equilibrium type" solution for $P(k+1\,|k)$ can be obtained by setting $P(k+1\,|k+1) = P(k\,|k)$, assuming for short time intervals, terms such as $\underline{H}$ remain constant. The resulting equation is

$$0\Phi^{-1}(\Delta)[P-S]\Phi^{-1\,T}(\Delta) = P - P\underline{H}^T(\underline{H}PH^T + R)^{-1}\underline{H}P \tag{5}$$

Using $\Phi^{-1} = (I - A\Delta)$ and solving for the individual terms of P results in

$$P_{12} = ((\Delta q_w(h^2P_{11} + R))^{1/2}/k) = P_{21} \tag{6a}$$

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*Associate Professor, Dept. of Electrical Engineering; presently an Aero-Space Technologist with NASA Langley Research Center. Member AIAA.

†It has been assumed that altitude can be accurately measured independently and hence is treated as a known.

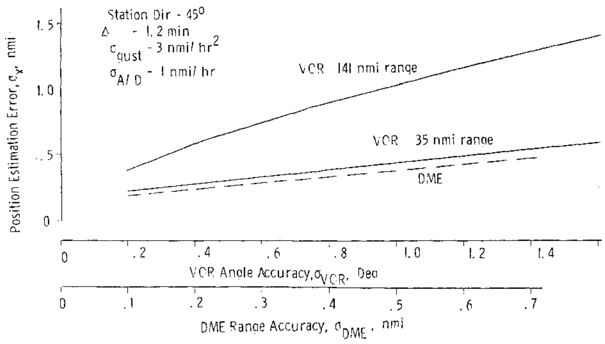

Fig. 1 Effect of observation errors.

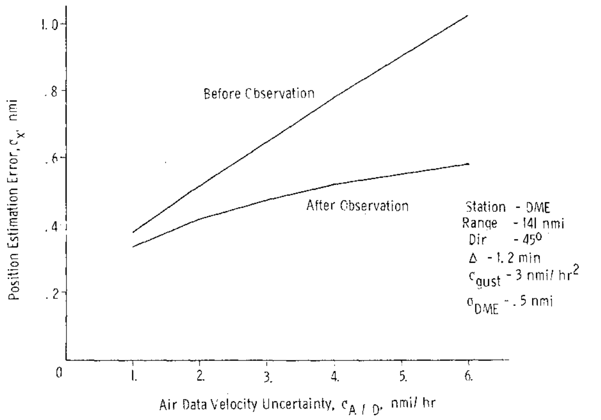

Fig. 2 Effect of air data velocity uncertainty.

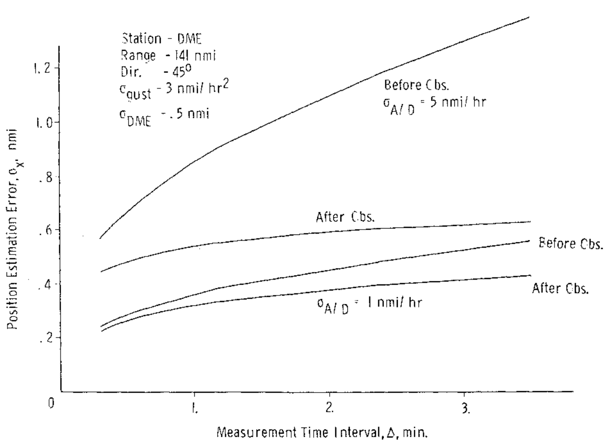

Fig. 3 Effect of time interval.

$$P_{22} = \frac{1}{\Delta}\left[\frac{hp_{11}(\Delta q_w)^{1/2}}{(h^2P_{11}+R)^{1/2}}\right] + \frac{\Delta q_w}{2}\times h^2P_{11}{}^2 - \frac{\Delta[\Delta q_w(h^2P_{11}+R)]^{1/2}(h^2P_{11}+2R)}{h} - \Delta(h^2P_{11}+R)\left(q_v - \frac{\Delta^2 q_w}{6}\right) = 0 \qquad (6b)$$

Discussion of Results

The solution of Eqs. (6) are shown in Figs. 1-3 for a range of parameters applicable to the area navigation problem. Figure 1 shows the effect of DME and VOR observation errors on position accuracy estimates for fixed air data errors, wind variations, and observation rate. For the DME system postulated in the 1977 time period of Ref. 1 [a DME RSS error of 0.51 nmi], an accuracy of 0.38 nmi is indicated. The effect of range is found to be insignificant. For example for a DME accuracy of 0.4 nmi at a ground range of 141 nmi, $(P_{11})^{1/2}=0.337$ nmi, while at 35 nmi, $(P_{11})^{1/2}=0.339$ nmi. The slight increase is attributed to the reduction in sensitivity due to the effect of slant range.

As is well known, VOR accuracy is a function of range, so two nominal ranges, 141 nmi and 35 nmi are shown on Fig. 1. For most situations, VOR accuracies are significantly worse than DME. For the 1977 period [VOR RSS accuracy of 2.12 nmi] a system accuracy of 1.54 nmi is obtained for a 35 nmi range and a 45° station location.

One can conclude that in an R/NAV system, making repeated observations where both DME and VOR stations are present and using optimal mean-square fitting (Kalman), the resultant accuracy would be determined primarily by the DME accuracy. The VOR observations would carry little weight.‡ Hence the total R/NAV, RSS accuracy attained when piloting errors (1 nmi) are included in the systems previously cited would be 1.07 nmi and 1.836 nmi for the DME and VOR only systems, respectively.

The system improvement available by filtering over the static single-position fix concept (Ref. 1), is seen by comparing the results at similar locations. For the static system, total accuracies of 1.4 nmi and 3.9 nmi occur for 35 nmi and 141 nmi ranges, respectively.[1] This compares with the 1.07 nmi error which is independent of range, for the DME system with filtering.

The study shows that significant changes in system accuracy occur with changes in air data accuracy. The effect of air data velocity variance is shown in Fig. 2 for both $[P(k+1|k)]^{1/2}$, the error immediately before an observation, and $[P_{11}(k+1|k+1)]^{1/2}$, the error immediately after, for nominal processing time and DME station accuracy. The effect of velocity uncertainty is most significant immediately before an observation where at 6 nmi/hr error, position uncertainties rise to 1 nmi. This effect is to be expected since the velocity uncertainty effects primarily the dead-reckoning calculation causing a cumulative effect over the time interval between measurements. Since air data are less accurate in the along-track mode (than cross-track), larger along-track errors occur and hence, larger separation is required.

In this analysis of system errors, the air data velocity has been resolved by coordinate transformation into along- and cross-track modes. Usually most of the velocity, and hence the error is along the desired track. During turns and maneuvers, however, the velocity vector may be only partially in the along-track direction. Hence, one must assume, based upon this study, that cross-track position uncertainty can deteriorate significantly during aircraft maneuvers. Further study of these flight modes is required before precise along- and cross-track route separation distances can be defined.

The effect of observation time intervals on system position uncertainty is shown in Fig. 3. Because the results of Fig. 2 indicate a coupling effect between air data accuracy and measurement interval time, both $\sigma_{A/D}=1$nmi/hr and $\sigma_{A/D}=5$nmi/hr curves are shown. At the lower velocity uncertainty, measurement time intervals appear to have much less of an effect although the separation, $[P_{11}(k+1|k)]^{1/2} - [P_{11}(k+1|k+1)]^{1/2}$ always increases as Δ increases. For $\sigma_{A/D}=5$ nmi/hr, measurement time intervals have a drastic effect. For example, at an observation interval change from 0.2 min to 3 min, the uncertainty increases from 0.6 to 1.3 nmi.

In many sophisticated R/NAV systems being built, fix rates shorter than 0.2 min are used. The information in Fig. 3 indicates that such fix rates do not significantly improve R/NAV accuracy. Also, when changing stations a 1-30 sec delay for lock-on may result in some DME receivers. Delays of this order should not seriously influence system accuracy.

‡In the filtering concept, the system fix is obtained in a manner significantly different from the geometric VOR/DME fix of Ref. 1. Resulting system accuracies, however, should be comparable.

Results indicate that gust deviations σ_{GUST} have only a second-order linear effect upon position accuracy. A linear curve fit shows that the gust slope coefficient is 0.0127 nmi/nmi/hr^2 over the range $0.5 < \sigma_{GUST} < 10.$ nmi/hr^2. Hence, a change in gust characteristics should have only small influence on accuracy. The situation is fortunate since wide ranges in gust velocity are encountered.

Conclusions

This paper develops a simple method for predicting R/NAV accuracy and for determining the effect of various system parameters. The development employs the statistical mean-square filtering equation and, hence, more realistically represents the accuracies attainable in R/NAV than the VOR/DME static geometric fixes used in Ref. 1.

The results indicate that, when present, DME measurements will almost solely determine the system accuracy. Any VOR measurement will be weighted very lightly in the filter unless their range is extremely close. For the 1977 period, using Ref. 1 figures, the pilot will be the major factor in system accuracy, when operating in the DME mode. For solely VOR stations to be useful in R/NAV, the system will have to be improved to accuracies significantly below 1.0° and locations less than 100 nmi in range.

The results indicate that the air data velocity uncertainties can influence the system accuracy significantly. It is this factor in the uncoupled analysis which distinguishes between the along- and cross-track system capability. This factor couples with the observation rate. One can improve accuracy by either reducing the velocity uncertainty or the observation interval or both. For the velocity uncertainties considered, $\sigma_{AD} < 5$ nmi/hr, measurement intervals $\Delta \simeq 1$ min provide reasonable along- and cross-track accuracies. The results also indicate that gusts variations seem to have little influence on the system accuracies.

The results demonstrate a simple method for analysis of the tradeoff features in R/NAV systems design. The capability to examine such design parameters independent of the detail system configuration should permit a better understanding of R/NAV requirements, a wider scope of R/NAV equipment for the diverse set of users, and an understanding of where improvement in future R/NAV systems can be most profitable.

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